MIMO BEAMFORMING AND RANDOM ACCESS

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Signal Directions & MIMO Beam Forming

- Goal
  - Instead of acquiring a large number of channel coefficients and then designing the signals based on them, one may acquire the *directions and amplitudes* of the path gains of the sparse multipath network directly.
  - Unlike phases, the directions of the multipath network change slowly with the movement of the transceivers or the scatterers and are invariant to frequencies, and thus can be estimated and learned from *uplink* training.
- Beam Network Strategy
  - Communicate via signal paths with direction and amplitude knowledge by inner and outer beams
- Case Studies
  - In this work, we illustrate such MIMO communication by a beam network methodology in two representative problems:
    1. The first problem is point-to-point MIMO communication.
    2. The second one is MIMO communication in an interference channel
  - We compare the performance of directly taking advantage of the partial CSIT without inner beamforming, and that of employing inner beamforming at the transmitters and receivers to reduce the dimensions of the equivalent channels, which include the inner transmit and receive beamforming, on which the outer beamforming can be optimized.

Channel Model

- MIMO Channel Matrix between two arrays

$$H = \underbrace{\left[\vec{b}_{D_{\mathrm{R}}}(\vec{q}_{\mathrm{R},k})\right]_{1,k}}_{B_{\mathrm{R}}} \underbrace{[a_k]_{k,k}}_{A} \underbrace{\left[\vec{b}_{D_{\mathrm{T}}}(\vec{q}_{\mathrm{T},k})\right]^{\dagger}_{1,k}}_{B_{\mathrm{T}}^{\dagger}}$$

  - $\vec{q}_R$ the incoming wave direction relative to a local basis
  - Delay due to the location $\vec{d}_i$ of the i-th array element affects the received signal by a multiplicative coefficient $e^{-j\frac{2\pi}{\lambda_c}\langle \vec{d}_i, \vec{q}_R\rangle}$.
  - Coefficients can be organized in the array response vector:

$$\vec{b}_D(\vec{q}_{\mathrm{R}}) = \left[e^{-j\frac{2\pi}{\lambda_c}\langle \vec{d}_i, \vec{q}_{\mathrm{R}}\rangle}\right]_{i=1:\#\mathrm{column}(D)}$$

  - Array response vector for the outgoing wave for transmit direction $\vec{q}_T$ can be obtained as $\vec{b}_D(\vec{q}_T)^{\dagger}$
- Received signal at receiver l is

$$\vec{\boldsymbol{y}}_l = \sum_k H_{l,k}\vec{\boldsymbol{x}}_k + \vec{\boldsymbol{w}}_l$$

Problem Formulation and Solutions
A. *Single-User MIMO Communication*

- Setting
  - In the classic single-user MIMO communication problem, there is one transmitter, one receiver.
  - The transmitter has knowledge of the signal paths' beam directions $\vec{q}_{T,k}, \vec{q}_{R,k}$, and the amplitudes of the path gains.
- Perfect CSIT Solution
  - If there is perfect channel state information at the transmitter (CSIT), one just designs the covariance matrix of the transmit signal $\Sigma = FF^{\dagger}$, or equivalently, its beamforming matrix F to maximize the data rate:

$$\mathcal{I}(\Sigma, H, \Omega) = \log\left(\det\left(I + H\Sigma H^{\dagger}\Omega^{-1}\right)\right)$$

Subject to the power constraint $Tr(\Sigma) \le \rho$.

- Since we do not have the perfect CSIT, a reasonable objective is to maximize the expected rate or ergodic rate:

$$\mathrm{E}\left[\mathcal{I}\left(\Sigma, B_{\mathrm{R}}\boldsymbol{A}B_{\mathrm{T}}^{\dagger}, I\right)\right]$$

Problem Formulation and Solutions
A. *Single-User MIMO Communication*

- Dimension Reduction by Inner and Outer Tx and Rx Beamforming
  - Let the beamforming matrix F to be a product of an inner beamforming matrix F_I and an outer beamforming matrix F_I, i.e., $F = F_I\, F_o$.
  - Let $QR(C)$ be the Q matrix from the thin QR decomposition of C.
  - We choose the inner transmit beamforming matrix to be $F_I = Q_R(B_T)$.
  - We choose the inner receive beamforming matrix to be $G_I^{\dagger} = Q_R(B_R)^{\dagger}$.
- After the inner beamforming, the received signal model becomes:

$$\vec{\boldsymbol{y}}_{\mathrm{e}} = G_{\mathrm{I}}^{\dagger}\vec{\boldsymbol{y}} = \underbrace{G_{\mathrm{I}}^{\dagger}HF_{\mathrm{I}}}_{H_{\mathrm{e}}}F_{\mathrm{O}}\vec{s} + \underbrace{G_{\mathrm{I}}^{\dagger}\vec{\boldsymbol{w}}}_{\vec{\boldsymbol{w}}_{\mathrm{e}}},$$

- Thus, the dimensions of the equivalent channel are reduced and, depending on the number of significant signal paths, could be much less than the number of antennas.

Problem Formulation and Solutions
A. *Single-User MIMO Communication*

- Suboptimal Algorithm
  - Since maximizing the ergodic rate is hard, we solve an alternative problem. We use the partial CSIT to calculate an approximate equivalent channel:

$$\hat{H}_{\mathrm{e}} = G_{\mathrm{I}}^{\dagger}B_{\mathrm{R}}|A|B_{\mathrm{T}}^{\dagger}F_{\mathrm{I}}$$

  - Then, we can design the outer covariance matrix $\Sigma_o = F_oF_o^{\dagger}$ or outer precoding matrix F_o by maximizing the non-ergodic rate with the same power constraint:

$$\max_{\Sigma_{\mathrm{O}}} \mathcal{I}\left(\Sigma_{\mathrm{O}}, \hat{H}_{\mathrm{e}}, I\right)$$
$$\text{s.t.}\quad \mathrm{Tr}\left(F_{\mathrm{I}}\Sigma_{\mathrm{O}}F_{\mathrm{I}}^{\dagger}\right) = \mathrm{Tr}\left(\Sigma_{\mathrm{O}}F_{\mathrm{I}}^{\dagger}F_{\mathrm{I}}\right) = \mathrm{Tr}\left(\Sigma_{\mathrm{O}}\right) \le \rho$$

- The solution $\overline{\Sigma}_O$ from the water-filling is asymptotically optimal when the numbers of transmit and receive antennas grow to infinity because the channel becomes parallel channels.

Problem Formulation and Solutions
B. *Communication in the MIMO Interference Channel*

- Setting
  - In a interference channel, the l-th transmitter has a private message to the l-th receiver. Each transmitter has knowledge of the beam directions and the amplitudes of the corresponding path gains.
- Perfect CSIT Solution
  - If there is perfect CSIT, one may design the covariance matrices $\Sigma_l = F_lF_l^{\dagger}$ or the beamforming matrices F_l to maximize the sum rate:

$$\max_{\Sigma_l\text{'s}} \sum_l \mathcal{I}(\Sigma_l, H_l, \Omega_l)$$
$$\text{s.t.}\quad \sum_l \mathrm{Tr}(\Sigma_l) \le \rho,$$

  - where the interference is treated as noise, and thus the interference plus noise covariance is:

$$\Omega_l = I + \sum_{k:k\neq l} H_{l,k}\Sigma_k H_{l,k}^{\dagger}.$$

Problem Formulation and Solutions
B. *Communication in the MIMO Interference Channel*

- Suboptimality of Narrow Inner Beam
  - While using transmit *narrow* inner beams $Q_R(B_{T,k,k})$ that only point to the desired user has no loss of optimality for the single user case, it is suboptimal for the interference channels
  - because the signal outside the desired user's subspace could cancel the interference in the undesired user's subspace.
- Optimal Inner Tx Beam
  - An *optimal* transmit inner beams is $F_{l,k} = Q_R([B_{T,k,k}]_{1,l})$, i.e., the orthonormal column basis of the subspaces to the desired and undesired users.
- Optimal Inner Rx Beam
  - An optimal receive inner beams is $G_{I,l}^{\dagger} = Q_R([B_{R,l,k}]_{1,k})^{\dagger}$, i.e., the orthonormal column basis of the subspaces from the desired and undesired transmitters.
- Dimension Reduction
  - As in the single user case, when the numbers of significant paths are less than the numbers of transmit and receive antennas, the equivalent channels achieve a reduction of dimensions.

Problem Formulation and Solutions
B. *Communication in the MIMO Interference Channel*

- Equivalent Channel
  - The equivalent channels constructed from the partial CSIT are

$$\hat{H}_{\mathrm{e},l,k} = G_{\mathrm{I},l}^{\dagger}B_{\mathrm{R},l,k}|A_{l,k}|B_{\mathrm{T},l,k}^{\dagger}F_{\mathrm{I},k}$$

- Suboptimal Algorithm
  - We attack the rate maximization problem of the equivalent channel,

$$\max_{\Sigma_{\mathrm{O}}\text{'s}} \sum_l \mathcal{I}\left(\Sigma_{\mathrm{O},l}, \hat{H}_{\mathrm{e},l,l}, \hat{\Omega}_{\mathrm{e},l}\right)$$
$$\text{s.t.}\quad \sum_l \mathrm{Tr}(\Sigma_{\mathrm{O},l}) \le \rho,$$

where

$$\hat{\Omega}_{\mathrm{e},l} = I + \sum_{k:k\neq l} \hat{H}_{\mathrm{e},l,k}\Sigma_{\mathrm{O},k}\hat{H}_{\mathrm{e},l,k}^{\dagger}$$

  - Using the dual link algorithm, we obtain a locally optimal solution $\overline{\Sigma}_{o,l}$ for the equivalent channel and calculate $\Sigma_l = F_{I,l}\overline{\Sigma}_{o,l}F_{I,l}^{\dagger}$.

Numerical Results
A. Single-User MIMO Communication

- Setting
  - We compare the achievable rates of 4 cases:
    (1) CSIT without inner beams, i.e., $F_{I,l} = I$,
    (2) CSIT with inner beams FI and GI , given in Section III,
    (3) and (4) are the same as (1) and (2) except that the CSIT is replaced with partial CSIT
  - The number of transmit and receive antennas are assumed to be 4x2, 8x4, 16x8.

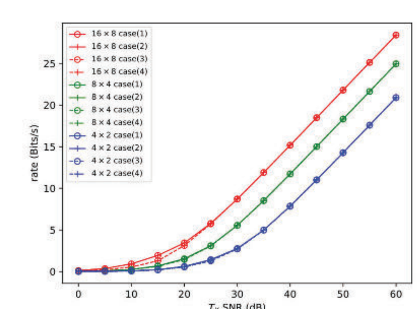
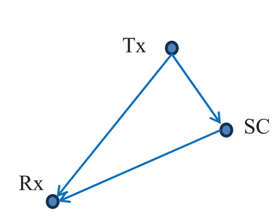


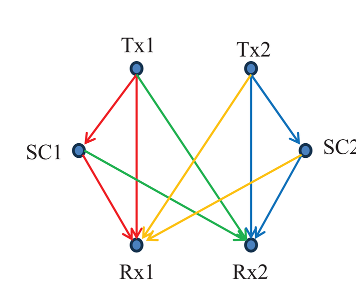
Fig. 1. A comparison of the achievable rates of the cases with CSIT or partial CSIT, with inner beams or without inner beams: single user MIMO channel

- Observation
  - We observe that, employing the beam network method, the partial CSIT has almost no effect on the performance compared to perfect CSIT.



Numerical Results
B. Communication in the MIMO Interference Channel

- Setting
  - For interference channels, we compare the rates of 6 cases:
    (1) CSIT without inner beams, i.e., $F_{I,l} = I$.
    (2) CSIT with optimal inner beams
    (3) CSIT with narrow inner beams that only point to the desired user
    (4), (5), (6) are the same as (1), (2), (3) except that the CSIT is replaced with partial CSIT.
  - The dual link algorithm is employed for all cases to design the covariance matrices, finding the locally maximal sum rate subject to the sum power constraint.
  - The numbers of transmit and receive antennas are assumed to be 4x2, 8x4, or 16x8 for both pairs of transmitter and receiver.



Numerical Results
B. Communication in the MIMO Interference Channel

- Observation
  - As expected, the optimal inner beam cases have the same performance as the no-beam cases.
  - We observe that as the numbers of antennas increase, the performances of partial CSIT without inner beams or with optimal inner beams approach those of perfect CSIT.
  - We also observe that the performances of narrow inner beams have smaller slopes than the ones without inner beams as the SNR increases due to the limited dimensions in making the interference orthogonal to the undesired user.
  - A compromise could be employing inner beams to cover the subspaces with significant path gains for the desired and undesired users.

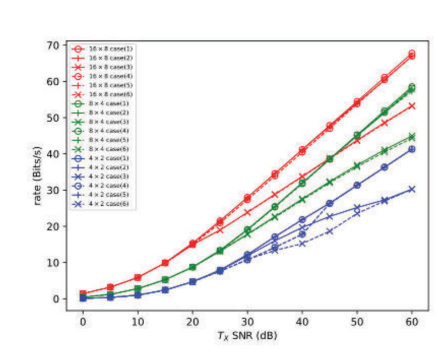


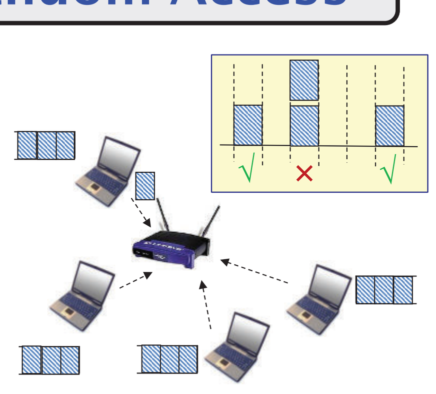
Fig. 2. A comparison of the sum rates: two user MIMO interference channels

Semi-unsourced Random Access

Random Access
Users are uncoordinated. No joint optimization
Receiver does not know who will transmit
Decoding or collision report

Semi-unsourced
Problem: Massive # of "potential users"
Small # of active users
Solution: Each user picks a temporary ID from a small size ID pool

Problem with the fully "unsourced" model
Receiver can't distinguish transmitters.
If multi. users transmit data streams, receiver can't distribute decoded data properly.



1. Manageable complexity of searching the ID pool
2. Low probability of multiple active users choosing the same ID

Multiple Coding Options at Each User

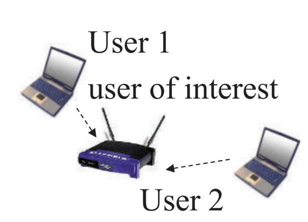
Each user has M coding options
$g_k \in \mathcal{G}_k,\ |\mathcal{G}_k| = M,\ g_k = \left[r_k, P_{X_k|g_k}\right]$
r_k : commun. rate. 2^{Nr_k} bits
$P_{X_k|g_k}$: input distribution
N: codeword length

Why multiple options?
No user coordination, opt. code unknown
To move advanced wireless capability to MAC layer, e.g. rates, power, beam
Support flexible adaptation at MAC layer

Assumption: User choose an arbitrary option without inform. others & receiver

System model: 3 users, multi-access
User 1: user of interest
Decode or collision report
User 2 : interfering user
Can decode if necessary
User 0: virtual user (e.g. channel state)
Can affect channel, nothing to decode

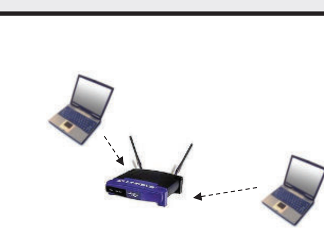
User 1
user of interest



User 2
Interfering user
User 0 (channel state)
virtual user

Random Access Communication Model

System Model
User 0: Determines channel state h
User 1: Chooses code g_1, message w_1, then send codeword $X_1^{(N)}$
User 2: Chooses code g_2, message w_2, then send codeword $X_2^{(N)}$
Receiver: Receives $Y^{(N)}$
Channel: $P(Y|X_1, X_2, h)$



coding vector
$\boldsymbol{g} = [g_1, g_2, h]$
Unknown to users and to receiver

Partition "coding vector" space into 4 regions: $R_1, R_{21}, \hat{R}, R_c$
For $\boldsymbol{g} \in R_1$, receiver intends to output $(\hat{g}_1, \hat{w}_1)$.
For $\boldsymbol{g} \in R_{21}$, receiver intends to output $(\hat{g}_2, \hat{w}_2)$ and $(\hat{g}_1, \hat{w}_1)$.
For $\boldsymbol{g} \in \hat{R}$, receiver intends to output $(\hat{g}_1, \hat{w}_1)$or "collision".
For $\boldsymbol{g} \in R_c$, receiver intends to output "collision".
Define $P_e(\boldsymbol{g})$ = prob. of erroneous output given $\boldsymbol{g}$.

Generalized error performance

$$\mathrm{GEP} = \sum_{\boldsymbol{g}} P_e(\boldsymbol{g})e^{-N\alpha_{\boldsymbol{g}}}$$

$$\left\{\alpha_{\boldsymbol{g}} \middle| \alpha_{\boldsymbol{g}} \ge 0, \sum_{\boldsymbol{g}} e^{-N\alpha_{\boldsymbol{g}}} = 1\right\}$$

Error Performance Bound

Decoder for user group *D*: Space to 3 regions, $R_D, \hat{R}_D, R_c$
For $\boldsymbol{g} \in R_D$, receiver intends to output $(\hat{\boldsymbol{g}}_D, \hat{\boldsymbol{w}}_D)$.
For $\boldsymbol{g} \in \hat{R}_D$, receiver to output $(\hat{\boldsymbol{g}}_D, \hat{\boldsymbol{w}}_D)$ or "collision".
For $\boldsymbol{g} \in R_c$, receiver intends to output "collision".
Define $P_{eD}(\boldsymbol{g})$ = prob. of erroneous output given $\boldsymbol{g}$.

$$\mathrm{GEP}_D = \sum_{\boldsymbol{g}} P_e(\boldsymbol{g})e^{-N\alpha_{\boldsymbol{g}}}$$

$$\left\{\alpha_{\boldsymbol{g}} \middle| \alpha_{\boldsymbol{g}} \ge 0, \sum_{\boldsymbol{g}} e^{-N\alpha_{\boldsymbol{g}}} = 1\right\}$$

$$\mathrm{GEP}_D \le \sum_{\boldsymbol{g}\in R_D}\left\{\sum_{S\subset D}\left[\sum_{\tilde{\boldsymbol{g}}\in R_D, \tilde{\boldsymbol{g}}_S = \boldsymbol{g}_S} P_{m[\boldsymbol{g},\tilde{\boldsymbol{g}},S]}e^{-N\alpha_{\boldsymbol{g}}} + \sum_{\tilde{\boldsymbol{g}}\notin R_D, \tilde{\boldsymbol{g}}_S = \boldsymbol{g}_S}\left(P_{t[\boldsymbol{g},\tilde{\boldsymbol{g}},S]}e^{-N\alpha_{\boldsymbol{g}}} + P_{i[\tilde{\boldsymbol{g}},\boldsymbol{g},S]}e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right)\right.\right.$$
$$\left.\left. + \sum_{\tilde{\boldsymbol{g}}\notin R_D\cup\hat{R}_D, \tilde{\boldsymbol{g}}_D = \boldsymbol{g}_D}\left(P_{t[\boldsymbol{g},\tilde{\boldsymbol{g}},D]}e^{-N\alpha_{\boldsymbol{g}}} + P_{i[\tilde{\boldsymbol{g}},\boldsymbol{g},D]}e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right)\right\}$$

$$P_{t[\boldsymbol{g},\tilde{\boldsymbol{g}},S]} = P\left\{P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}(\boldsymbol{w}_D), \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}} \le \gamma_{[\boldsymbol{g},\tilde{\boldsymbol{g}},S]}\right\} \text{ for } \boldsymbol{g}\in R_D, \tilde{\boldsymbol{g}}\notin R_D, \boldsymbol{g}_S = \tilde{\boldsymbol{g}}_S$$

$$P_{m[\boldsymbol{g},\tilde{\boldsymbol{g}},S]} = P\left\{\exists\tilde{\boldsymbol{w}}_D, (\tilde{\boldsymbol{w}}_D, \tilde{\boldsymbol{g}}) \overset{S}{=} (\boldsymbol{w}_D, \boldsymbol{g}),\right.$$
such that $P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}(\boldsymbol{w}_D), \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}}$
$$\left.\le P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\tilde{\boldsymbol{g}}_D}(\tilde{\boldsymbol{w}}_D), \tilde{\boldsymbol{g}}_{\bar{D}}\right)e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right\}$$
for $\boldsymbol{g}, \tilde{\boldsymbol{g}} \in R_D$ with $\boldsymbol{g}_S = \tilde{\boldsymbol{g}}_S$

$$P_{i[\tilde{\boldsymbol{g}},\boldsymbol{g},S]} = P\left\{\exists\boldsymbol{w}_D, (\tilde{\boldsymbol{w}}_D, \tilde{\boldsymbol{g}}) \overset{S}{=} (\boldsymbol{w}_D, \boldsymbol{g}),\right. \text{ such that}$$
$$\left.P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}(\boldsymbol{w}_D), \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}} > \gamma_{[\boldsymbol{g},\tilde{\boldsymbol{g}},S]}\right\}$$
for $\boldsymbol{g} \in R_D, \tilde{\boldsymbol{g}} \notin R_D, \boldsymbol{g}_S = \tilde{\boldsymbol{g}}_S$

Error Performance Bound

$$P_{m[\boldsymbol{g},\tilde{\boldsymbol{g}},S]} \le 2^{N\sum_{k\in D\setminus S} r_{\tilde{g}k}} e^{-N\alpha_{\boldsymbol{g}}} P\left\{\log\left[P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}, \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}}\right] \le \log\left[P\left(Y^{(N)} \mid \bar{\boldsymbol{X}}^{(N)}_{\tilde{\boldsymbol{g}}_D}, \tilde{\boldsymbol{g}}_{\bar{D}}\right)e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right]\right\}$$
for $\boldsymbol{g}, \tilde{\boldsymbol{g}} \in R_D$ with $\boldsymbol{g}_S = \tilde{\boldsymbol{g}}_S$

where $P\left\{\log\left[P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}, \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}}\right] \le \log\left[P\left(Y^{(N)} \mid \bar{\boldsymbol{X}}^{(N)}_{\tilde{\boldsymbol{g}}_D}, \tilde{\boldsymbol{g}}_{\bar{D}}\right)e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right]\right\}$ can be numerically evaluated using joint distribution $P_{\boldsymbol{g}}\left(\boldsymbol{X}^{(N)}\right)P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}\right)P_{\tilde{\boldsymbol{g}}}\left(\bar{\boldsymbol{X}}^{(N)}\right)$

For $\boldsymbol{g}, \tilde{\boldsymbol{g}} \in R_D$ with $\boldsymbol{g}_S = \tilde{\boldsymbol{g}}_S$, $\left(P_{t[\boldsymbol{g},\tilde{\boldsymbol{g}},S]}e^{-N\alpha_{\boldsymbol{g}}} + P_{i[\tilde{\boldsymbol{g}},\boldsymbol{g},S]}e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right) \le$

$$e^{-N\alpha_{\boldsymbol{g}}} P\left\{\log\left[P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}, \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}}\right] \le \log\left[2^{N\sum_{k\in D\setminus S} r_{gk}} P_{\tilde{\boldsymbol{g}}}\left(Y^{(N)}\right)e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right]\right\}$$
$$+2^{N\sum_{k\in D\setminus S} r_{gk}} e^{-N\alpha_{\tilde{\boldsymbol{g}}}} \bar{P}\left\{\log\left[P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}_{\boldsymbol{g}_D}, \boldsymbol{g}_{\bar{D}}\right)e^{-N\alpha_{\boldsymbol{g}}}\right] > \log\left[2^{N\sum_{k\in D\setminus S} r_{gk}} P_{\tilde{\boldsymbol{g}}}\left(Y^{(N)}\right)e^{-N\alpha_{\tilde{\boldsymbol{g}}}}\right]\right\}$$

where $P\{\quad\}$ can be numerically evaluated using joint distribution $P_{\boldsymbol{g}}\left(\boldsymbol{X}^{(N)}\right)P\left(Y^{(N)} \mid \boldsymbol{X}^{(N)}\right)$, while $\bar{P}\{\quad\}$ can be evaluated using $P_{\boldsymbol{g}}\left(\boldsymbol{X}^{(N)}\right)P_{\boldsymbol{g}}\left(Y^{(N)}\right)$

Conclusion

- Signal directions and MIMO beamforming
  - Proposed the beam network methodology for MIMO communications when the numbers of antennas are large and the channel is sparse.
  - investigated two representative problems to demonstrate the simplicity and performance of the methodology.
  - Showed that the performance of the beam network method approaches that of perfect CSIT in single-user MIMO channels and interference MIMO channels
- Semi-unsourced random access
  - Proposed a semi-unsourced random access communication model
  - Supported multiple coding options at each transmitter
  - Obtained new achievable error performance bound