Binary Signal Alignment: Optimal Solution is Polynomial-time and Linear-time Solution is Quasi-optimal

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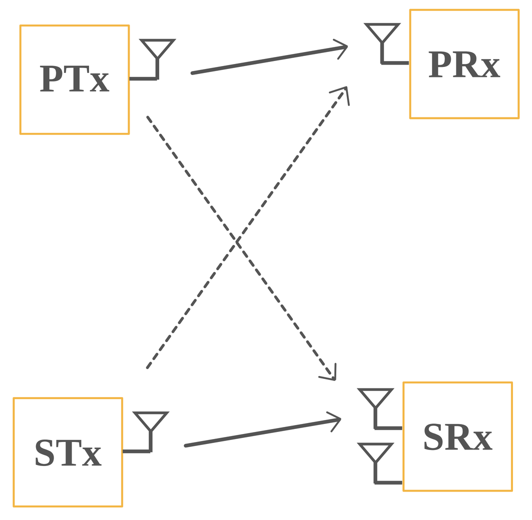
Problem Statement

- Examining a fundamental scenario within the context of underlay CRN, involving a **strong** primary and a **weak** secondary user.



- PTx and STx are equipped with a single antenna.
- SRx is equipped with $M_s \geq 2$.

Motivation

- **Goal**: Reliably decode the low power underlay user in the presence of strong interference.
- **Prior art**:
  - **Traditional methods**: Need of accurate cross-channel estimates and SINR $\geq \tau$.
  - **Sophisticated methods**: Secondary decoding with simple repetition coding **w/o** requiring **CSI** at extremely low SINR [1].
- **This work**: Shows that the formulation in [1] can be solved **optimally** for binary vectors in **polynomial time**, where its two-step **linear-time** solution is **quasi-optimal**.

[1] Ibrahim, Karakasis, Sidiropoulos. "A Simple and Practical Underlay Scheme for Short-range Secondary Communication", IEEE TWC, June 2022

System Model

- The received signal at the SRx is:

$$\mathbf{Y}_s = \sqrt{\alpha_s}\mathbf{x}_s\mathbf{h}_s + \sqrt{\alpha_p}\mathbf{x}_p\mathbf{h}_{ps} + \mathbf{W}_s, \quad (1)$$

- $\mathbf{h}_{ps} \in \mathbb{C}^{1\times M_s} \rightarrow$ channel responses between PTx and SRx.
- $\mathbf{h}_s \in \mathbb{C}^{1\times M_s} \rightarrow$ channel responses between STx and SRx.
- $\mathbf{x}_s \in \mathbb{C}^{N\times 1}$ and $\mathbf{x}_p \in \mathbb{C}^{N\times 1}$ transmitted signals by STx and PTx
- $\mathbf{W}_s \in \mathbb{C}^{N\times M_s} \rightarrow$ AWGN with i.i.d. elements $\sim \mathcal{CN}(0, N_0)$.

Objective: Reliably decode secondary signal $\mathbf{x}_s$ given the received signal $\mathbf{Y}_s$.

Transmission Protocol

- The STx **repeats** its information twice at very low power -much lower than PTx-.

$$\mathbf{x}_s = \left[\mathbf{s}^\top, \mathbf{s}^\top\right]^\top, \mathbf{s} \in \mathbb{C}^{\frac{N}{2}\times 1}. \quad (2)$$

- The PTx **does not** repeat, and its information can be partitioned as,

$$\mathbf{x}_p = \left[\mathbf{p}_1^\top, \mathbf{p}_2^\top\right]^\top, \mathbf{p}_1, \mathbf{p}_2 \in \mathbb{C}^{\frac{N}{2}\times 1}. \quad (3)$$

Objective: Decode the repeated $\mathbf{s}$ at extremely **low SINR** given the received signal.

Signal Detection via CCA

- Split the signal at SRx to obtain the signal views $\mathbf{Y}_1$, $\mathbf{Y}_2$,

$$\mathbf{Y}_s = \begin{bmatrix}\mathbf{Y}_1\\ \mathbf{Y}_2\end{bmatrix}, \ \mathbf{Y}_r = \mathbf{s}\mathbf{h}_s + \mathbf{p}_r\mathbf{h}_{ps} + \mathbf{W}_r, \quad r \in \{1, 2\}$$

- MAX-VAR $\rightarrow$ common component $\mathbf{g}$ that the views share.

$$\min_{\mathbf{g}, \{\mathbf{q}_r\}_{r=1}^2} \sum_{r=1}^{2} \|\mathbf{Y}_r\mathbf{q}_r - \mathbf{g}\|_2^2 \quad \text{s.t.} \quad \|\mathbf{g}\|_2^2 = 1. \quad (4)$$

- $\{\mathbf{q}_r\}_{r=1}^2 \in \mathbb{C}^{M_s\times 1}$ are the dimensionality-reducing operators.
- In the noiseless case $\mathbf{g}^\star = \lambda\mathbf{s}$, $\lambda \in \mathbb{C}$ [1].

[1] Ibrahim, Karakasis, Sidiropoulos. "A Simple and Practical Underlay Scheme for Short-range Secondary Communication", IEEE TWC, June 2022

Reformulation of CCA

- Reformulate MAX-VAR to the following eigenvector problem,

$$\max_{\mathbf{g}} \mathbf{g}^{\mathcal{H}} \underbrace{\left(\sum_{r=1}^{2} \mathbf{Y}_r\mathbf{Y}_r^\dagger\right)}_{\mathbf{A}\in\mathbb{C}^{N/2\times N/2}} \mathbf{g}, \quad \text{s.t.} \quad \|\mathbf{g}\|_2^2 = 1, \quad (5)$$

- After letting $\mathbf{Y}_r = \mathbf{U}_r\mathbf{\Sigma}_r\mathbf{V}_r^{\mathcal{H}}$, (5) reduces to the principal component problem

$$\max_{\mathbf{g}} \left\|\mathbf{g}^{\mathcal{H}} \underbrace{[\mathbf{U}_1, \mathbf{U}_2]}_{\mathbf{M}\in\mathbb{C}^{N/2\times 2M_s}}\right\|_2^2 \quad \text{s.t.} \quad \|\mathbf{g}\|_2^2 = 1. \quad (6)$$

- Overall complexity is bounded by $\mathcal{O}\left(NM_s^2\right)$ via truncated SVD.

Binary CCA

- If $\mathbf{s} \in \{\pm 1\}^{N/2}$ and the receiver demodulates both (I/Q) components, then the eigenvector problem can be written as

$$\max_{\mathbf{g}} \mathbf{g}^\top \Re\{\mathbf{A}\}\mathbf{g}, \quad \text{s.t.} \quad \|\mathbf{g}\|_2^2 = 1. \quad (7)$$

- Then the equivalent principal component problem can be solved again with $\mathcal{O}\left(NM_b^2\right)$,

$$\max_{\mathbf{g}} \left\|\mathbf{g}^{\mathcal{H}} \underbrace{[\Re\{\mathbf{U}_1\}, \Re\{\mathbf{U}_2\}, \Im\{\mathbf{U}_1\}, \Im\{\mathbf{U}_2\}]}_{\mathbf{M}\in\mathbb{R}^{N/2\times 4M_s}}\right\|_2^2 \quad \text{s.t.} \quad \|\mathbf{g}\|_2^2 = 1. \quad (8)$$

Optimal Solution - BPSK

- if $\mathbf{s} \in \{\pm 1\}^{N/2}$ we can pose the eigenvector problem as

$$\max_{\mathbf{g}|\mathbf{g}\in\{\pm 1\}^{N/2}} \mathbf{g}^\top \Re\{\mathbf{A}\}\mathbf{g}, \quad (11)$$

- $\mathbf{A}$ is Hermitian $\rightarrow \Re(\mathbf{A})$ symmetric PSD.
- In the noiseless case rank $(\mathbf{A}) = 3 \rightarrow$ rank $(\Re(\mathbf{A})) \leq 5$.
- In the presence of noise rank $\nearrow$, ... truncation to effective rank through spectral factorization.
- In general, the complexity to solve (9) grows exponentially, but $\Re(\mathbf{A})$ is rank deficient of rank $D \Rightarrow \mathcal{O}\left(N^D\right)$.

[2] Karystinos, Liavas. "Efficient Computation of the Binary Vector That Maximizes a Rank-Deficient Quadratic Form", IEEE T. INFORM. THEORY, June 2010.

Optimal Solution - 4QAM

- if $\mathbf{s}$ contains 4QAM symbols, the eigenvector problem can be reformulated as,

$$\max_{\mathbf{b}|\mathbf{b}\in\{\pm 1\}^N} \mathbf{b}^\top\mathbf{H}\mathbf{b}, \quad (9)$$

$$\mathbf{b} = \left[\Re\{\mathbf{g}\}^\top, \Im\{\mathbf{g}^\top\}\right]^\top, \quad \mathbf{b} \in \mathbb{R}^{N\times 1},$$
$$\mathbf{H} = \begin{bmatrix}\Re\{\mathbf{A}\} & -\Im\{\mathbf{A}\}\\ \Im\{\mathbf{A}\} & \Re\{\mathbf{A}\}\end{bmatrix}, \quad \mathbf{H} \in \mathbb{R}^{N\times N}. \quad (10)$$

- $\mathbf{H}$ is symmetric PSD with rank $(\mathbf{H}) = 6$ in the noiseless case.
- Noisy regime $\rightarrow$ truncate to rank 6.
- Problem (10) again can be solved with $\mathcal{O}\left(N^D\right)$, because $\mathbf{H}$ is rank deficient of rank D [2].

[2] Karystinos, Liavas. "Efficient Computation of the Binary Vector That Maximizes a Rank-Deficient Quadratic Form", IEEE T. INFORM. THEORY, June 2010.

Conclusions

- Simple repetition coding and multiple receiver antennas allows CCA decoding and enables seamless coexistence of users in the same wireless medium.
- Directly incorporating binary constraint into CCA-based signal alignment for BPSK and 4QAM constellations.
- Suboptimal two-step solution approaches optimal performance with lower complexity, making it practical for real-world applications.

Numerical Results

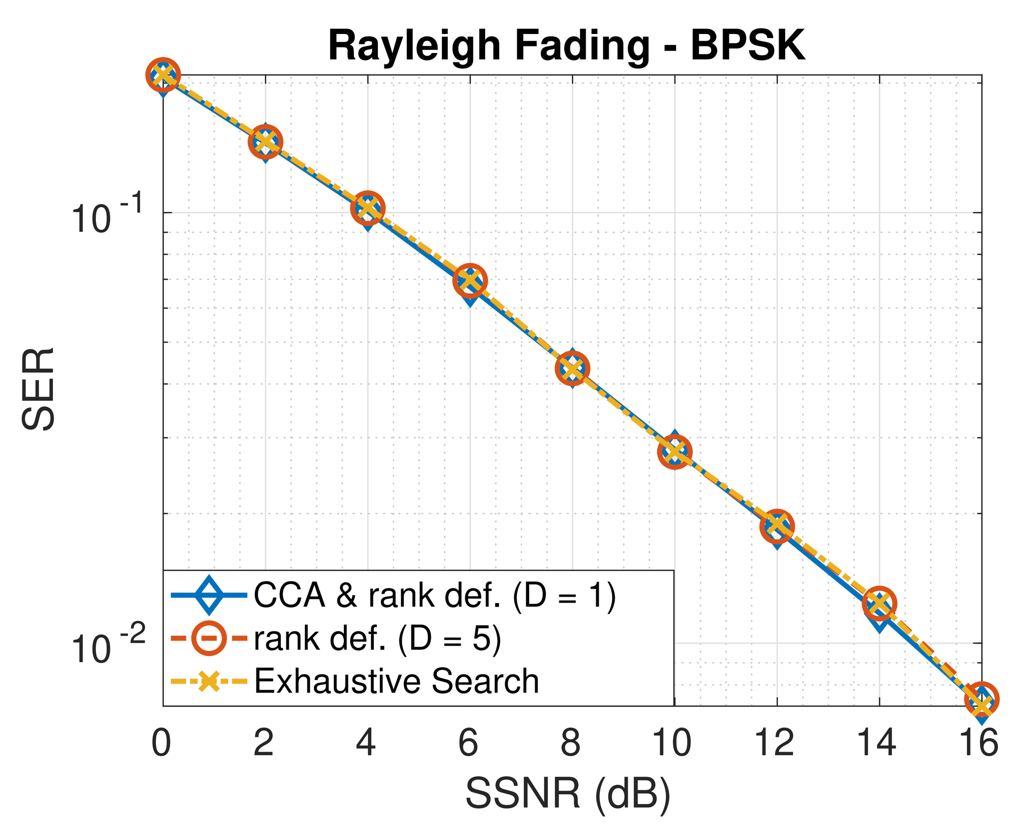


Figure 1: Secondary user performance - BPSK modulation ($N = 32$). SSINR is fixed to -45 dB. Primary sends QPSK.

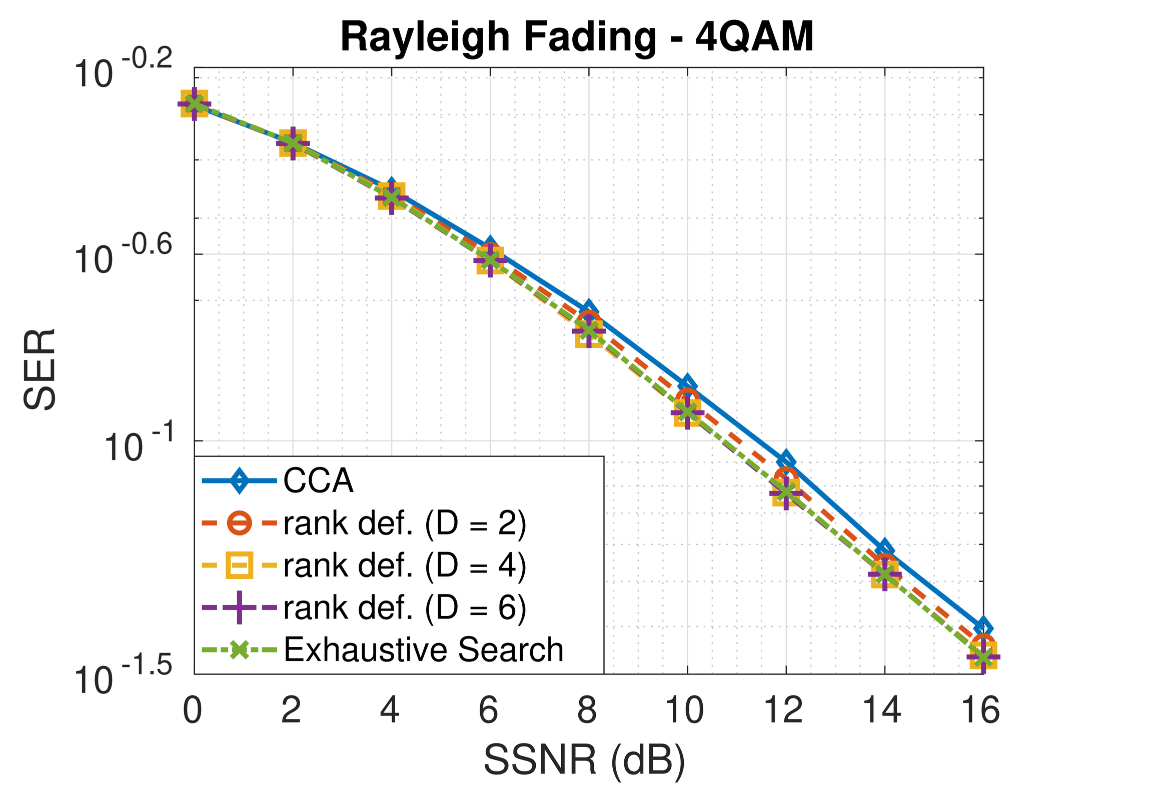


Figure 2: Secondary user performance - 4QAM modulation ($N = 16$). SSINR is fixed to -45 dB. Primary sends QPSK.